СВОЙСТВА ПЛАЗМЫ С БОЗЕ КОНДЕНСАТОМ ЗАРЯЖЕННЫХ ЧАСТИЦ

А.Д. Долгов

НГУ, Новосибирск 90
ИТЭФ, 117218, Москва
University of Ferrara, Ferrara 40100, Italy

XII Марковские чтения

14 мая, 2014 года

Quite unusual results
found in our papers:
A.D., A. Lepidi, G. Piccinelli,
JCAP 0902 (2009) 027; Phys. Rev D, 80 (2009) 125009; JCAP 08 (2010) 031;
A.D.,A. Lepidi Phys.Lett. A375(2011) 3188.
Similar results but by another method:
G. Gabadadze, R.A. Rosen,
Phys. Lett. B 658 (2008) 266;
JCAP 0810 (2008) 030;
JCAP 1004 (2010) 028.

Textbook formula for screening:

$$U(r) = \frac{Q}{4\pi r} \to \frac{Q\ \exp(-m_D r)}{4\pi r},$$

because the time-time component of the photon propagator acquires "mass":

$$k^2 \to k^2 + \Pi_{00}(k) = k^2 + m_D^2\,,$$

where e.g. for relativistic fermions

$$m_D^2 = e^2 \left(T^2/3 + \mu^2/\pi^2\right).$$

In presence of charged particle condensate the screening is not exponential but power law and oscillating as a function of distance. We did not publish our work for about half a year, but then found out that an oscillating screening is known for plasma with degenerate fermions and is observed in experiment - Friedel oscillations. Physics is different but qualitative behavior is the same.

Strangely until recently the effects on screening from condensate of a charged Bose field were not well studied, though it is a textbook problem.

Consider electrically neutral plasma with large electric charge density of fermions compensated by charged bosons. Bosons condense when their chemical potential reaches maximum value:

$$\mu_B = m_B .$$

Otherwise it is impossible to make larger asymmetry between bosons and antibosons.

Equilibrium distribution of condensed boson

$$f_B^C = C\delta^{(3)}(\mathrm{q}) + \frac{1}{\exp\left[(E - m_B)/T\right] \pm 1}$$

is a solution of the kinetic equation, it annihilates the collision integral for an arbitrary constant C.

f_{eq} is always determined by two parameters, either T and μ, or T and C, iff $\mu = m_B$.

Collision integral:

$$I_{coll} \sim |A_{fi}|^2 \Pi f_f \Pi (1 \pm f_i) - (inverse)$$

If T-invariance holds, i.e. $|A_{if}| = |A'_{fi}|$:

$$I_{coll} \sim \left[\Pi f_i (1 \pm f_f) - (i \leftrightarrow f) \right] d\tau \, .$$

$I_{coll} = 0$ for arbitrary T and C
iff $\mu = m$.

If T-invariance is broken and $|A_{if}| \neq |A'_{fi}|$, :

$$I_{coll}[f_{eq}] \sim \Pi f_i (1 \pm f_f) \left[|A_{fi}|^2 - |A_{if}|^2 \right]$$

This term is surely non-vanishing!

Do equilibrium distibutions remain the same in T-broken theory?

Breaking of T-invariance is unobservsble if only one reaction channel is open. In this case $T_{if} = T^*_{fi}$ with time reflected momenta.
f^C_B annihilates collision integral after summa
over all relevant processes, due to S-matirix
unitarity or CPT and conservation of probab

Instead of the detailed balance condition
there operates "the cyclic balance" condition

Screening properetie s of medium are express
through f which is not necessarily equiibrium
one. In calculations neither imaginary time
method which may be inconvenient in
presence of condensate or out of equiibroum,
nor Matsubara-Keldysh technique are used.
We started from the quantum equations
of motion, solved them with Green's function
up to e^2 order, and averaged corresponding
operators not only over vacuum but also
over "non-empty" medium.

Operator Maxwell equations:

$$\partial_\nu F^{\mu\nu}(x) = \mathcal{J}_B^\mu(x) + \mathcal{J}_F^\mu(x)\,,$$

where bosonic current is

$$\mathcal{J}_B^\mu(x) = -i\,e[(\phi^\dagger(x)\partial^\mu\phi(x)) - (\partial^\mu\phi^\dagger(x))\phi(x)] + 2e^2 A^\mu(x)|\phi(x)|^2\,,$$

plus fermionic current:

$$\mathcal{J}_F^\mu(x) = e\bar{\psi}\gamma_\mu\psi\,.$$

Using equation of motion for quantum operator ϕ:

$$(\partial^2 + m^2)\phi(x) = \mathcal{J}_\phi(x)$$

express ϕ through A_μ:

$$\phi(x) = \phi_0(x) + \int d^4y \, G_B(x-y)\mathcal{J}_\phi(y) \, ,$$

ϕ_0 is free field operator. In the lowest order in e take $\phi = \phi_0$ in $\mathcal{J}^\mu_B(x)$.

The r.h.s. of the Maxwell equations in e^2 order is linear (but non-local) in A_μ and bilinear in ϕ_0 and ψ_0.

Expand free fields as usually:

$$\phi_0(x) = \int d\tilde{q}\left[a(q)e^{-iqx} + b^\dagger(q)e^{iqx}\right].$$

Average over medium:

$$\langle a^\dagger(\mathrm{q})a(\mathrm{q}')\rangle = f_B(E_q)\delta^{(3)}(\mathrm{q}-\mathrm{q}'),$$
$$\langle a(\mathrm{q})a^\dagger(\mathrm{q}')\rangle = [1+f_B(E_p)]\delta^{(3)}(\mathrm{q}-\mathrm{q}').$$

Unity is subtracted, since it is vacuum contribution.

The Fourier transform of the Maxwell equations in plasma is:

$$\left[k^2 g^{\mu\nu} - k^{\mu}k^{\nu} + \Pi^{\mu\nu}(k)\right] A_{\nu}(k) = \mathcal{J}^{\mu}(k)$$

where the boson contribution is:

$$\Pi^{B}_{\mu\nu}(k) = e^2 \int \frac{d^3q}{2(2\pi)^3 E} \left[f_B(E,\mu) + \bar{f}_B(E,\bar{\mu})\right] \left[\frac{l_\mu l_\nu}{l^2 - m^2} + \frac{p_\mu p_\nu}{(p^2 - m^2} - 2g_{\mu\nu}\right]$$

where $l = k + q$, $p = k - q$, and $E = \sqrt{q^2 + m^2}$.

Solving Fourier transformed the linear Maxwell equation for A_t:

$$\Pi_{tt}(0,k) = \frac{e^2}{2\pi^2}\int_0^\infty \frac{dq\, q^2}{E_B}[f_B(E_B,\mu_B)$$

$$+\bar{f}_B(E_B,\bar{\mu}_B)][1+\frac{E_B^2}{kq}\ln|\frac{2q+k}{2q-k}|]\,,$$

plus similar contribution from fermions which neutralize the plasma.

This is the well known result for Π_{tt} in order e^2.

The screened Coulomb potential is the Fourier transform of tt-component of the photon Green's function in medium:

$$U(r) = e^2 \int \frac{d^3k}{(2\pi)^3} \frac{e^{ikr}}{k^2 + \Pi_{tt}(k)} = \frac{e^2}{2\pi^2 r} \int_0^\infty \frac{dkk \, \sin kr}{k^2 + \Pi_{tt}}.$$

Asymptotics of the potential of charged impurities is determined by the singularities of Π_{tt} in complex k-plane.

Comment.

Singularities of $f(z)$:

$$f(z) = \int_a^b dy g(z, y)$$

in complex z-plane appear at such z for which singularites of $g(z, y)$, i.e. $y_c(z)$, in complex y-plane coinsides with the bounds of integration, a or b, or $y_c(z)$ pinches the contour of integration.

Two types of singularities:

1. Poles of $[k^2 + \Pi_{tt}(k)]^{-1}$. E.g. Debye pole. Necessary to check that the position of the poles are at small k, such that the infrared asymptotics of Π_{tt} is valid.

2. Singularities of $\Pi_{tt}(k)$, originating from the pinch of the integration contour in q-plane by poles of f and by branch points of log.

Without condensate one obtains the usual k-independent Debye screening:

$$\Pi_{tt}(0, k) = m_D^2$$

originating from a pole at imaginary axis of k.

With condensate the corrections to Π_{tt} at low k are infrared singular:

$$\frac{\Delta\Pi_{tt}}{e^2} = \frac{m_B^2 T}{2k} + \frac{C}{(2\pi)^3 m_B}\left(1 + \frac{4m_B^2}{k^2}\right)$$

Both terms in the r.h.s. appear only if $\mu = m_B$.

Instead of exponential the screening becomes
power law and oscillating, depending upon
parameters, m_j:

$$\Pi_{tt} = m_0^2 + m_1^3/k + m_2^4/k^2.$$

May this have something to do with confinem
Recent paper: P. Gaete, E. Spalucci, 0902.00
– confinement in Higgs phase.

Contribution from poles in the limit of large $m_2 r$ but when power law terms are subdominant:

$$U(r)_{pole} = \frac{Q}{4\pi r}\exp\left(-\sqrt{e/2}\,m_2 r\right)\times \cos\left(\sqrt{e/2}\,m_2 r\right).$$

Oscillating screening is known for degenerate fermions - Friedel oscillations. Observed in experiment.

The screening electrons are waves with $k = k_F$ (from B. Shklovsky).

Comment.

Friedel oscillations are commonly believed to be zero T phenomenon, because in this case the integral over q is in finite interval and the singularity in k appears when log branch point coincides with the upper limit of the integration.

However the "pinch"method works at $T \neq 0$ and the $T = 0$ limit can be recovered by summing all the singularities. Non-zero T corrections, absent in textbooks can be obtained in this way.

Contribution from the integral along imagina axis is nonzero because Π_{00} contains an odd in k term. If $m_2 \neq 0$, the dominant term is

$$U(r) = -\frac{12Qm_1^3}{\pi^2 e^2 r^6 m_2^8}.$$

If $T \neq 0$, $\mu = m_B$, but the condensate is not yet formed, the asymptotic decrease of the potential becomes:

$$U(r) = -\frac{Q}{\pi^2 e^2 r^4 m_1^3} = -\frac{2Q}{\pi^2 e^2 r^4 m_B^2 T}.$$

Contribution from logarithmic cuts (analogo
to Friedel oscillations for fermions).

If the first "pinch" (between the poles of $f(q)$ and logarithmic branch point) dominates:

$$U_1(r) = -\frac{32\pi Q}{e^2 m_B r^2} \frac{e^{-z}}{\ln^2(2\sqrt{2}z)} \sin z \, ,$$

where $z = 2r\sqrt{2\pi T m_B}$.
NB: $U_1(r)$ is inversely proportional to e^2 and formally vanishes at $T \to 0$, but remains finite if $\sqrt{T m_B} r \neq 0$.

All pinches are comparable:

$$U(r) \approx -\frac{3Q}{2e^2T^2m_B^3r^6\ln^3(\sqrt{8m_BT}r)}.$$

$U \sim T^{-2}$ valid if $r \ll 1/\sqrt{16\pi T m_B}$, i.e. if $T = 0.1$K and $m_B = 1GeV$ the distance should be bounded from above as $r \ll 3 \cdot 10^{-8}$ cm.

Condensation of vector bosons.

$W^\pm$ would condense in the early universe if lepton asymmetry was sufficiently high.

It leads to large electric asymmetry of W, such that $\mu_W = m_W$.

Plasma neutrality was maintained by quarks and leptons.

Vector bosons have additional degrees of freedom, their spin states, and their condensation demonstrates richer possibilities: Depending on the sign of the pairwise spin-spin coupling W's would condense either in $S = 0$ (scalar) state or in $S = 2$ (ferromagnetic) state.

Magnetic spin-spin interaction through one photon exchange (similar to Breit equation):

$$U_{em}^{spin}(r) = \frac{e^2\rho^2}{4\pi m_W^2}\Big[\frac{(S_1\cdot S_2)}{r^3} - 3\,\frac{(S_1\cdot r)(S_2\cdot r)}{r^5} - \frac{8\pi}{3}(S_1\cdot S_2)\delta^{(3)}(r)\Big].$$

Here ρ is the ratio of magnetic moment of W to the standard one.

For S-wave the energy is shifted by the last term only.

Local quartic self-coupling of W:

$$U_{4W}^{(spin)} = \frac{e^2}{8m_W^2 \sin^2\theta_W}(S_1 S_2)\delta^{(3)}(r).$$

The net result $U_{em} + U_{4W}$ is negative, so $S = 2$ state is energetically favorable and spontaneous magnetization in the early universe is possible.

Suppression due to screening.
The ij component of W propagator probably remains massless: $\Pi_{ij} \sim 1/q^2$. In QED it is true in perturbation theory, while in non-Abelian theories the screening may occur in higher orders of perturbation theory due to infrared singularities. The screening would diminish the long-range ferromagnetic spin-spin coupling while the local W^4 coupling is not screened.

If the propagator is modified, and the wave function of $\boldsymbol{W}$-bosons is constant in space, the spin-spin energy shift is:

$$\delta E \sim \int \frac{d^3 q \delta(q)}{(2\pi)^3} \frac{q^2 (S_1 S_2) - (\mathrm{q}\mathrm{S}_1)(\mathrm{q}\mathrm{S}_2)}{q^2 + \Pi_{ss}(\mathrm{q})}$$

$\delta E = 0$, if $\Pi_{ss} \neq 0$ at $q = 0$.

However, the integration over space should be done with an upper limit, l, equal to the average distance between the W bosons so instead of $\delta^{(3)}(q)$, we obtain:

$$\int_0^l d^3 r e^{iqr} = \frac{4\pi}{q^3}\left[\sin(ql) - ql\cos(ql)\right].$$

and the energy shift is non-zero:

$$\delta E = -\frac{(S_1 S_2)e^2}{l^3 m_W^2} F(l)\,,$$

$$F(l) = \int_0^\infty \frac{dx\,\left[x\ \sin x + l^2 \Pi_{ss}\cos x\right]}{x^2 + l^2 \Pi_{ss}(x/l)}.$$

If $l^2\Pi_{ss}$ is nonnegligible the e.m. part of the spin-spin interaction would be suppresse and the ferromagnet turns into an antiferrom This might happen at T above the EW phase transition when the Higgs condensate is destroyed and $m_{W,Z}$ appear as a result of temperature and density corrections and are relatively small.

The quantitative statement depends upon the (unknown) modification of the space-space part of the photon propagator in presence of the Bose condensate of charged $\boldsymbol{W}$ – a problem to solve.

Problem of large scale magnetic fields: $B \sim \mu G$ at several kpc. In the intergalactic space the fields are probably 2-3 orders of magnitude weaker, but still non-vanishing

Dynamo operates only in galaxies.

Maybe ferromagnetism of W might create seeds for large scale magnetic fields.

Screening of magnetic fields is connected with the space-space components, which, in the homogeneous and isotropic case is

$$\Pi_{ij} = a(k)\left(\delta_{ij} - \frac{k_i k_j}{\mathrm{k}^2}\right) + b(k)\frac{k_i k_j}{\mathrm{k}^2}\,.$$

Multiplying Π_{ij} by δ_{ij} and by $k_i k_j$ we obtain $b(k) = 0$ and

$$a(k) = \frac{e^2}{32\pi^3}\int \frac{d^3q}{E}\,(f + \bar{f}) \left[2 + \frac{2k^2(4q^2 - k^2)}{4(\tilde{\mathrm{k}}\tilde{\mathrm{q}})^2 - k^4}\right].$$

where $\vec{k}\vec{q} = kq\cos\theta$.

If only the condensate term is retained:

$$a^{(C)}(k) = \frac{e^2 C}{8\pi^3 m_B} \equiv e^2 m_C^2 .$$

Since $a(0) = const \neq 0$, the magnetic field is exponentially screened. In absence of magnetic monopoles magnetic field can be screened only by currents, hence plasma with BEC of electrically charged Bose field must be superconductive- well known result. (Two regimes of superconductivity: weakly coupled Cooper pairs, i.e. BCS or strong coupling BEC regime.)

If $\mu < m_B$, then $\Pi_{ij}(k)$ vanishes as k^2 in the limit $k \to 0$, as expected:

$$a(k) \approx \frac{e^2 k^2}{24\pi^2} \int \frac{dq}{E} (f + \bar{f})$$

and magnetic fields are not screened.

If $\mu = m_B$, even without condensate, i.e. at $C = 0$, $a(k)$ vanishes only as a first power of k, which leads to unusual screening features.

$a(k)$ is singular in the limit $m_B = 0$, since the integral diverges as $1/q^2$ at the lower limit of integration, $q = 0$. Moreover, a singularity at $k = 0$ exists for massive particles if $\mu = m_B$. The singularity comes from the integration region where $q \sim k$ due to singularity of $f(q)$ at low q. So we obtain for $k \to 0$:

$$a^{(sing)}(k) = \frac{e^2 T}{16} k\,.$$

For small k this term would dominate over the usual k^2 term and change the screening behavior.

In the transverse gauge, $k_j A_j = 0$, the Maxwell equation can be solved as

$$A_i(x) = \int d^3y \, G(x-y) \, \mathcal{J}_i(y) \, .$$

The asymptotics of $G(r)$ at large r is determined by

$$G(r) = \frac{(-i)}{4\pi^2 r} \int_0^\infty \frac{dkk \left(e^{ikr} - e^{-ikr}\right)}{k^2 + a(k)} \, .$$

$a(k)$ may contain odd terms in k, so the integral along the half real k-axis cannot be extended to the whole real axis.

It leads to non-canonical screening terms.

Since $a(k) = k^2 + e^2 m_C^2 + e^2 T k/16$, the integral can be rewritten as:

$$G(r) = \frac{(-i)}{4\pi^2 r} \int_0^\infty dkk \left(e^{ikr} - e^{-ikr}\right) \frac{(k^2 + e^2 m_C^2 - e^2 T k/16)}{(k^2 + e^2 m_C^2)^2 - e^4 T^2 k^2/256}.$$

The integral of the even part is expressed through the residues of the poles in the complex k-plane at:

$$k^{(pole)} = \pm i \sqrt{e^2 m_C^2 - \frac{e^4 T^2}{1024}} \pm \frac{e^2 T}{32}.$$

If $m_C > e^2T/32$, the screened potential would be exponentially cut with superimpose oscillations. For $e^2T \ll 32\,m_C$, the Green function takes the form:

$$G(r) \sim \exp\left(-em_C r\right) \cos\left(e^2 rT/32\right).$$

In this case the spatial damping scale is much shorter than the oscillation scale. However, for $eT \sim m_C$ the scales are comparable.

The contour of the integration of odd in $\boldsymbol{k}$, part can be closed in upper or lower quadrant of the complex k-plane. So in addition to the poles in these quadrants the contributions from the integrals over the imaginary axis are to be included. They produce a power law screening.

If $\boldsymbol{C = 0}$, but $\boldsymbol{\mu = m_B}$, then at small $\boldsymbol{k}$: $\boldsymbol{a(k) \approx e^2 kT/16}$, so the Green's function drops as: $\boldsymbol{G(r) \sim 8/(\pi^2 e^2 r^2 T)}$. This is realized when $\boldsymbol{r > 1/T}$.

In presence of condensate the Green's function acquires an additional constant term $e^2 m_C^2$. In this case the contribution of the integral over the imaginary axis of k gives $G \sim T/(16\, e^2 \pi^2 r^4 m_C^4)$.
Changing of the asymptotics of screening signals formation of the condensate.

THE END

Calculation of singularity.

It is convenient to separate the integral into two parts $0 < q < k/2$ and $k/2 < q < \infty$. In the first part we introduce the new integration variable $x = 2q/k$, so $0 < x < 1$. In the limit of small k the energy can be expanded as $E_B \approx m_B + k^2x^2/8m_B$. At small q the distribution function is infrared singular:

$$\left[\exp\left(\frac{E_B - m_B}{T}\right) - 1\right]^{-1} \approx \frac{2m_BT}{q^2} = \frac{8m_BT}{k^2x^2} (.$$

Usually this singularity is not dangerous because it is canceled by the integration measure, $\sim q^2$. However, the logarithmic term behaves as k/q for $q > k$ and as q/k for $q < k$. Thus the integral is finite, but it does not vanish as k^2 when $k \to 0$.

The first part of the integral with $q < k/2$ can be taken analytically and we obtain:

$$a_1^{(s)}(k) = \frac{e^2kT}{8\pi^2} \int_0^1 dx \left[2 - \left(x - \frac{1}{x}\right) \ln\left|\frac{1+x}{1-x}\right|\right.$$

$$\frac{e^2kT}{8\pi^2}\left(1+\frac{\pi^2}{4}\right).$$

There is also another contribution coming from the part of the integral with $q > k/2$. As $k \to 0$, the second part of the integral, $k/2 < q < \infty$, gives:

$$a_2^{(s)}(k) = \frac{e^2kT}{8\pi^2}\int_1^{\infty} dx\left[2-\left(x-\frac{1}{x}\right)\ln\left|\frac{1+x}{1-x}\right.\right.$$

$$\frac{e^2kT}{8\pi^2}\left(-1+\frac{\pi^2}{4}\right),$$

such that the total contribution is:

$$a^{(s)}(k) = a_1^{(s)}(k) + a_2^{(s)}(k) = \frac{e^2T}{16}\,k\,. \quad (4)$$

For small k this term could dominate over the usual k^2 term and would change the screening behavior.

so we present the denominator as half of sum and difference of even and odd function as following:

$$f(k) = [f(k) + f(-k)]/2 + [f(k) - f(-k)]/2 \quad (5)$$

Since $a(k) = k^2 + e^2 m_C^2 + e^2 Tk/16$, eq. (??) can be rewritten as:

$$G(r) = \frac{(-i)}{4\pi^2 r} \int_0^\infty dkk \frac{\left(e^{ikr} - e^{-ikr}\right)\left(k^2 + e^2 m_C^2 - e^2 Tk/16\right)}{(k^2 + e^2 m_C^2)^2 - e^4 T^2 k^2/256} \quad (6)$$

The integral of the even part may be transformed, as usually, into the integral along the whole real axis and after closing the contour in the upper (for e^{ikr}) or lower (for e^{-ikr}) half-plane we express the result through the residues in the corresponding poles in the complex k-plane at:

$$k^{(pole)} = \pm i \sqrt{e^2 m_C^2 - \frac{e^4 T^2}{1024}} \pm \frac{e^2 T}{32}. \quad (7)$$

If $m_C > e^2T/32$, the resulting screened potential would be exponentially cut with superimposed oscillations. For $e^2T \ll 32\, m_C$, the Green function takes the form:

$$G(r) \sim \exp(-em_C r)\, \cos(e^2 rT/32)\,. \qquad (8)$$

In this case the spatial damping scale is much shorter than the oscillation scale. However, if $eT \sim m_C$ the scales are comparab