

Sterile neutrinos, cosmology and modified gravity

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Additional light component

Neutrino masses with the Einstein gravity

$f(R)$ gravity

Dark energy in $f(R)$ gravity

Anomalous growth of perturbations

Sterile neutrino and $f(R)$ gravity

Conclusions

Based on:

1. H. Motohashi, A. A. Starobinsky and J. Yokoyama, Prog. Theor. Phys. **124**, 541 (2010) — standard neutrinos + $f(R)$ gravity .
2. H. Motohashi, A. A. Starobinsky and J. Yokoyama, Phys. Rev. Lett. **110**, 121302 (2013) — standard + sterile neutrinos + $f(R)$ gravity.

Amount of light component

Light – (practically) massless at the time of BBN and recombination.

It is conventionally expressed in terms of effective neutrino types:

$$\rho_r = \rho_\gamma \left[1 + N_{\text{eff}} \frac{7}{8} \left(\frac{4}{11} \right)^{4/3} \right]$$

with $N = 3.046$ for the standard three flavor neutrino species.

BBN (Izotov and Thuan, 2010): $N_{\text{eff}} = 3.7_{-0.7}^{+0.8}$ (95%).

From recent CMB and other observational data (P. A. R. Ade et al., arXiv:1303.5076):

$$N_{\text{eff}} = 3.36_{-0.64}^{+0.68} \quad (95\%; \text{Planck+WP+highL})$$

$$N_{\text{eff}} = 3.62_{-0.48}^{+0.50} \quad (95\%; \text{Planck+WP+highL+}H_0)$$

$$N_{\text{eff}} = 3.52_{-0.45}^{+0.48} \quad (95\%; \text{Planck+WP+highL+}H_0\text{+BAO})$$

Conclusion: one additional (sterile) neutrino species is well possible.

Cosmological upper limit on neutrino masses

(Absence of) damping of the power spectrum of density perturbations at small scales.

For three standard neutrino species:

$$\sum_i m_{\nu i} < 0.23 \text{ eV} \quad (95\%; \text{Planck+WP+highL+BAO})$$

For larger number of species the upper limit becomes weaker but not too much. In the case of one additional thermal sterile neutrino and 3 usual neutrinos with $\sum_{i=1}^3 m_{\nu i} \approx 0.06 \text{ eV}$,

$$N_{\text{eff}} < 3.80, \quad m_{\nu, \text{ster}}^{\text{eff}} < 0.42 \text{ eV} \\ (95\%; \text{Planck+WP+highL+BAO})$$

Conclusion: in the scope of the Einstein gravity, it is not possible to have a sterile neutrino with the standard number density and the restmass $m \gtrsim 0.5 \text{ eV}$.

Why interest in one sterile neutrino?

Anomalies in some ground experiments though with marginal statistical significance:

1. The MiniBoone anomaly.
2. Gallium anomaly in the SAGE and GALLEX experiments.
3. Reactor anomalies.

If confirmed, their explanation requires a fourth (sterile) neutrino with the restmass $m \sim 1 \text{ eV}$.

Will this destroy cosmology completely?

$f(R)$ gravity

The simplest model of modified gravity considered as a phenomenological macroscopic theory in the fully non-linear regime and non-perturbative regime. It can produce models of present geometrical dark energy alternative to a cosmological constant.

$$S = \frac{1}{16\pi G} \int f(R) \sqrt{-g} d^4x + S_m$$

$$f(R) = R + F(R), \quad R \equiv R^\mu_{\mu}$$

Field equations

$$\frac{1}{8\pi G} \left(R^\nu_\mu - \frac{1}{2} \delta^\nu_\mu R \right) = - \left(T^\nu_{\mu(vis)} + T^\nu_{\mu(DM)} + T^\nu_{\mu(DE)} \right) ,$$

where $G = G_0 = \text{const}$ is the Newton gravitational constant measured in laboratory and the effective energy-momentum tensor of DE is

$$8\pi G T^\nu_{\mu(DE)} = F'(R) R^\nu_\mu - \frac{1}{2} F(R) \delta^\nu_\mu + (\nabla_\mu \nabla^\nu - \delta^\nu_\mu \nabla_\gamma \nabla^\gamma) F'(R) .$$

Because of the need to describe DE, de Sitter solutions in the absence of matter are of special interest. They are given by the roots $R = R_{ds}$ of the algebraic equation

$$Rf'(R) = 2f(R) .$$

Degrees of freedom

I. In quantum language: particle content.

1. **Graviton** – spin 2, massless, transverse traceless.

2. **Scalaron** – spin 0, massive, mass - R -dependent:

$$m_s^2(R) = \frac{1}{3f''(R)} \text{ in the WKB-regime.}$$

II. Equivalently, in classical language: number of free functions of spatial coordinates at an initial Cauchy hypersurface.

Six, instead of four for GR – two additional functions describe massive scalar waves.

Thus, $f(R)$ gravity is a **non-perturbative** generalization of GR. It is equivalent to scalar-tensor gravity with $\omega_{BD} = 0$ (if $f''(R) \neq 0$).

Models of present dark energy in $f(R)$ gravity

Much more difficult to construct than inflationary models (the $R + R^2$ model and close ones).

An example of the viable model satisfying viability conditions in the present Universe ([Starobinsky, 2007](#)):

$$f(R) = R + \lambda R_0 \left(\frac{1}{\left(1 + \frac{R^2}{R_0^2}\right)^n} - 1 \right)$$

with $n \geq 2$. $f(0) = 0$ is put by hand to avoid the appearance of a cosmological constant in the flat space-time.

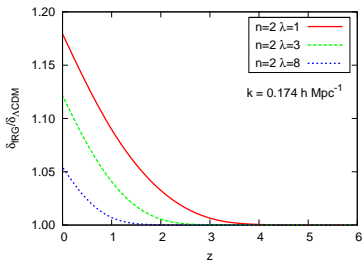
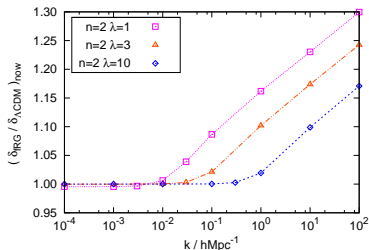
Similar models: [Hu and Sawicki, 2007](#); [Appleby and Battye, 2007](#).

No good microscopic justification for both the energy scale and the complicated form of $f(R)$ needed ($0 < f' < 1$).

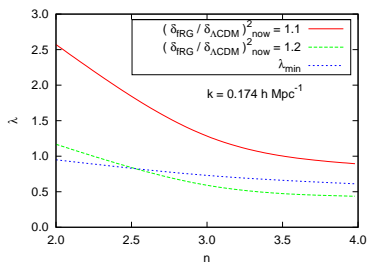
Anomalous growth of perturbations in $f(R)$ gravity

Occurs deeply in the sub-horizon regime when the scalaron becomes light (its Compton length exceeds the comoving scale of perturbations):

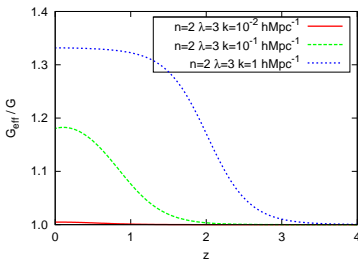
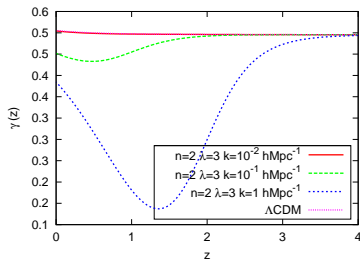
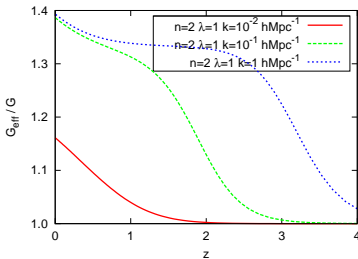
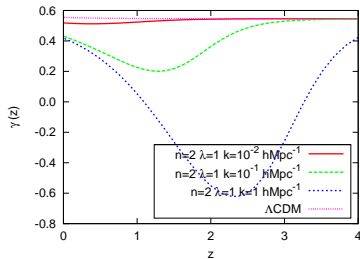
$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G_{\text{eff}}\rho\delta = 0, \quad G_{\text{eff}} = \frac{G}{f'} \frac{1 + 4\frac{k^2}{a^2}\frac{f''}{f'}}{1 + 3\frac{k^2}{a^2}\frac{f''}{f'}}.$$



Constraints in the parameter space

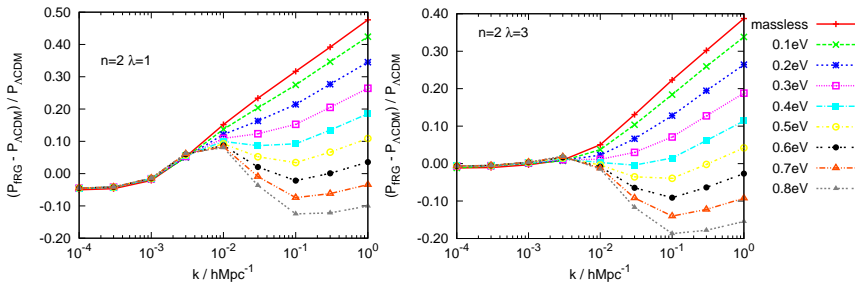


Evolution of $\gamma(z)$ and $G_{\text{eff}}(z)/G$



Massive neutrinos with $f(R)$ gravity

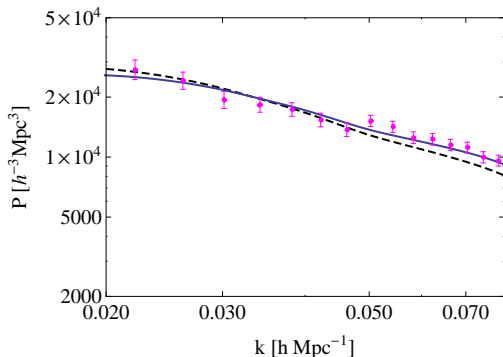
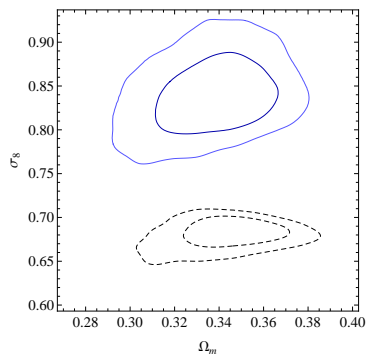
The anomalous growth of perturbations may be partially compensated by an increase of $\sum_{\nu} m_{\nu}$ as compared to the standard Λ CDM, up to $\mathcal{O}(0.5 \text{ eV})$ in the case of 3 standard neutrinos.



More interesting results for one additional sterile neutrino (assumed to be significantly more massive than the standard ones).

Sterile neutrino and $f(R)$ gravity

If the mass of the 4th sterile neutrino is fixed to **1 eV**:



$$\chi^2_{\Lambda\text{CDM}} - \chi^2_{fRG} = 9.55$$

(one more additional parameter compared to the standard ΛCDM model).

Conclusions

- ▶ Standard cosmology based on the Einstein gravity admits the existence of one additional (sterile) neutrino type with the standard number density, but its mass should be less than 0.5 eV .
- ▶ Cosmology based on $f(R)$ gravity admits one massive sterile neutrino with the mass $m_{\nu,\text{ster}} \sim 1 \text{ eV}$, and for a fixed mass $m_{\nu,\text{ster}} = 1 \text{ eV}$, fits cosmological observational data significantly better than the standard ΛCDM model.
- ▶ This sterile neutrino mass range is very interesting for ground-based neutrino experiments.